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Research article

A solution to the transportation hazard problem in a supply chain with an unreliable manufacturer

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Abstract: The current study focuses on a two-echelon supply chain for a reliable retailer, an unreliable manufacturer, and selling price-dependent demand. Due to an unreliable manufacturer and transportation hazards, shortages arise, which negatively impact the reputation of the retailer. Moreover, customers are more conscious of the environment, as a result, most of the industry focuses on the production of green products. To reduce the holding cost of the retailer, a fuel consumption-based single-setup-multi-unequal-increasing-delivery policy was utilized in this current study. With this transportation policy, the number of shipments increases, which directly increases carbon emissions and transportation hazards. To protect the environment, the green level of the product is enhanced through some investments. The demand varies with the price of the product as well as with the level of the greenness of the product. Due to uncertain demand, the rate of the production is treated as controllable. A classical optimization technique and distribution-free approach have been utilized to obtain the optimum solution and the optimized system profit. To prove the applicability, the study is illustrated numerically and graphically via a well-explained analysis of sensitivity. The study proves that single-setup-multi-unequal-increasing delivery policy is 0.62% beneficial compared to single-setup-single-delivery policy and 0.35% better than the single-setup-multi-delivery policy.

Keywords: supply chain management; unreliability; transportation; single-setup-multiple-unequal-delivery-policy; green quality

1. Introduction

Supply chain management (SCM) is an invisible rope that tides the participants of that Supply Chain (SC) strongly. It ensures the smooth conduction of the SC and maximizes its total profit or minimizes its total cost. A vendor-buyer model of SC was first studied by Goyal [1]. Nowadays, the unreliability of the players in any SC plays a critical role in optimizing the cost or profit. Unreliability may occur due to certain circumstances, which can increase the profit of the system. Simultaneously, demand cannot always be constant [2]. It may depend on various factors, like quality, price [3], advertisement of the product availability [4], service [5], and many other factors. In this study, the demand for the product varies according to two factors, namely the price of the product and the green level of the product. Until now several SC models were studied in the literature by considering the demand variability based on the selling price of the product [6] or the level of the greenness of the product [7]. However, the corresponding demand variabilities have yet to be reported in the literature along with smart technologies. In general, the players of any SC will be treated as reliable. However, this cannot always be true in these current circumstances. The players of the SC may maximize their profits by providing some wrong information, which means that some unreliability will arise in the SC. Due to the unreliability of the manufacturer, the retailer faces a shortage situation, which damages their reputation and the reduces the demand for the product. Thus, the industry manager needs to prevent the unreliability problem in the SC. There are many types of unreliability in SCs, such as unreliable manufacture [8], unreliable retailer [9], unreliable customer [10], unreliable supplier [11], unreliable information and channels [12], unreliable manufacturing system [13]. The manufacturer of this study is unreliable and hides the number of products from the retailer.

An important part of a SC is transportation. The strategy of the production of the whole ordered quantity at a single time reduces the manufacturer's production cost but delivering the whole produced amount in a single lot increases the holding cost (HC) of the retailer. Usually, retailers are situated in a place where the HC is greater than the manufacturer [10]. That is why it is profitable for an SC to transport the entirety of the produced items in different lots. The sizes of the lots may be equal [14] or unequal [15] depending upon the demand for the product. In this study, a single-setup-multi-unequal-increasing (SSMUID) strategy was applied to reduce the cost of holding of the retailer.

The number of transportation of the manufactured products in SC is increased for the application of the SSMUID policy. Due to transportation hazards and the SSMUID strategy, a huge amount of carbon emission occurs during transportation. To reduce carbon emission and keep the environment clean from carbon, the consumed fuel-dependent transportation and carbon emission costs are utilized [16]. Green products always help to keep our environment pollution-free. Regarding the environmental concerns, an investment was introduced to increase the green level of the product [7]. Lead time is another major issue in running an SC smoothly. Each industry maximized its profit by reducing the lead time. This study considers an unreliable manufacturer along with transportation hazards that occurs during transportation, which leads to a shortage in the SC. This study is concerned about this lead time by reducing the transportation hazard.

1.1. Research gaps

Based on the above discussion, the following research gaps were found and supplemented by the present model.

- SSMUID transportation policy was introduced by Hota et al. [5]; however, they neglected the concept of a transportation hazard, which is very important for any SSMUID transportation strategy.
- The Green level of any product always increases the demand for the product [7]. However, the concept of unreliability and consideration of the green level have yet to be presented in the literature.
- Regarding the literature, several studies have been conducted to solve the unreliability issue [11, 17]. But, unreliability in the SC along with the consideration of the green level of the product, has not been not considered in any existing literature.

1.2. Contribution

The above-mentioned research gaps were realized and solved in this study. Thus, the main contributions of this study are as follows:

- The current study focuses on resolving the problem of transportation hazards along with the problem of an unreliable manufacturer.
- In this model, one unreliable manufacturer, one reliable retailer and a particular type of product with a variable demand are formulated.
- The retailer faces shortage problems due to the unreliability of the manufacturer and transportation hazards.
- The shortage problem is handled by introducing hazards cost, utilizing smart manufacturing and applying variable backorder price discounts.
- Environmental sustainability was achieved by applying fuel-dependent carbon emission costs and investing in the green product.

A brief literature review along with an author's contribution table is given in Section 2. A list of used symbols, the problem, and the presumptions for the study are described in Section 3. The model and the methodology for solving it are respectively described in Section 4 and Section 5. A numerical justification of the model with sensitivity and graphs, is presented in Section 6. Finally, the managerial insights are illustrated in Section 7 and the conclusion and differences in the future are described in Section 8.

2. Literature review

In this section, the details of the literature gap and existing studies are discussed.

2.1. SC and unreliability

SCM is the practice of coordinating the necessary activities of the manufacture and retailer for services to the end customers. There are many studies on SCM. Omair et al. [18] studied the advancement of a choice for the prioritization of the suppliers on sustainability components. They provided a platform for the manufacturer to better understand the capability and established that the suppliers have to continue working with the manufacturer for sustainable SCM. Ullah et al. [19] established an ideal remanufacturing methodology and reusable bundling capacity beneath the stochastic request and return rate for a closed-loop SCM.

Currently, a problem affecting the smooth functioning of an SC is unreliability. It is very essential for the industry managers to prevent the unreliability problem and conduct an SC smoothly. The manufacturer in this study is unreliable and produces less than the ordered processes by the retailer, which causes a shortage. Recently, Tayyab and Sarkar [20] developed a textile SCM by applying an interactive fuzzy programming approach. Hota et al. [5] studied unreliable manufacturers and solved the problem by applying backorder price discounts. Using RFID, the retailer's unreliability problem was solved by Sardar et al. [17] and Sardar and Sarkar [9]. Ullah and Sarkar [21] solved the unreliable information problem by using RFID. Applying macro prediction, Guo et al. [22] reduced the forecasting uncertainty in the SC by applying information sharing through which the robustness of the system may be reduced. In the study performed by Xiao and Xu [23], unreliability occurred for information asymmetry. In the study performed by Sarkar [24], the production system was unreliable also, Caŕdenas-Barroń et al. [25] utilized the reworking strategy for defective products, produced due to the unreliable production system. Recently, Dhahri et al. [26] applied the concept of transportation delay and prioritization rules for the delivery. Stochastic dynamic programming and simulation optimization were used by them to optimize the result. But, none of these studies considered an SSMUID policy for their model.

2.2. *Transportation strategies with transportation hazards*

When running an SC, transportation plays a vital role. Without proper transportation strategy, an cannot be run. There were several transportation modes to transport the product from the manufacturer's warehouse to the retailer's showroom [27]. Based on the transportation trips, several strategies were developed in previous studies. In general a single-setup-single-delivery (SSSD) transportation policy is used for the transportation of the product [28]. However, if the manufacturer starts the production after getting the order and transports the order quantity using a multiple delivery policy, that is a single-setup-multi-delivery (SSMD) transportation policy that reduces the HC of the retailer, then the total system cost will be reduced [4]. Sarkar et al. [29] studied the effect of improving the quality of production and reducing carbon emissions in a model of SC by using an SSMD policy. All of the existing models consider an equal number of products in each shipment. However, in reality, this is not always possible. Sometimes the manufacturer transports the product based on the demand, which may be increase or decrease. Thus the concept of unequal delivery shipments is very essential for transportation [15]. With this strategy, all of the produced items are shipped to the retailer in different lots with unequal lot sizes. There are some products for which the demand never decreases, and for those types of products Hota et al. [5] introduced the SSMUID policy. Compared to the other transportation policies, SSMUID provides a better result. Since the present model deals with some product like medicine, the demand for which never decreases, the SSMUID policy was adopted to develop this model. Since this study considers transportation using road vehicles, the disruption of transportation was a major issue for this study; additionally the present study assumes that hazards in transportation can occur randomly following a certain probability distribution. It is very common for the transportation system to face a hazard during shipment in an SC. That hazard sometimes seriously affects the SC in many ways. Mainly transportation hazards affect the lead time which, causes a shortage. It is quite natural for the transportation hazards to increase for the application of the SSMUID policy because the number of shipments increases in this policy. Therefore, to prevent the shortage problem, in this study, a cost was introduced. The manufacturer bears the transportation hazard cost, and it depends on the distance of hazards from the manufacturer. The distance at which the hazard occurs is a random variable that may follow any particular type of distribution. There is a research gap as there is no study that includes an unreliable manufacture, the SSMUID policy and green products in which a transportation hazard occurred. This study fills in the research gap by introducing transportation hazards to an SC model with an unreliable manufacturer and the SSMUID policy.

To date, different SC models with different strategies have been reported in the literature, however, the concept of SSMUID along with random transportation hazards, is not considered. In the present study, this gap was is addressed.

Table 1. Authors contribution table.

Author	Unreliable	SS	TH	GL	DDO	PR
Dey et al. [4]	NA	SSMD	NA	NA	advertisement	V
Hota et al. [5]	manufacturer	SSMUID	NA	NA	service	F
Sana [7]	NA	NA	NA	√	SP & GL	F
Guchhait et al. [10]	information	SSSD	NA	NA	NA	F
Park and Lee [11]	retailer	SSSD	NA	NA	NA	F
Chen et al. [12]	channel	NA	NA	NA	NA	F
Sardar et al. [17]	retailer	SSSD	NA	NA	service	F
Sarkar [24]	NA	SSSD	NA	NA	reliability	F
Dey et al. [30]	NA	SSMD	NA	NA	NA	V
Sana [31]	NA	NA	NA	present	SP, CEI	F
This model	manufacturer	SSMUID	present	present	SP & GL	V

Note: SS: Shipment strategy; TH: Transportation hazard; GL: Green level; PR: Production rate; DDO: Demand depends on; CEI: carbon emission index; NA: Not considered, SP: Selling price; F: Fixed; V: variable.

2.3. Green quality and green product

Nowadays, a challenging job for industry managers is integrating social and environmental issues in the SC. For this reason, in this twenty-first century, the related application and research continues to increase to address the changing and different concerns regarding the sufficient determination of buyers. Firms have applied numerous advanced and eco-friendly business policies that continue the progress. In the last two centuries, applications of nonrenewable energy for transportation systems and industries reduced our natural resources, which has caused harm to flora and fauna because greenhouse gases increase in the environment for due to overuse of these resources [32]. Countries have devoted their efforts to keeping the environment safe by decreasing the number of pollutants discharged by their activities and uses. As a result, the development and research section of the governments of every country is looking for possible paths that can be developed sustainably. Many issues are included in sustainable development like green technology, green product development, carbon emission reduction, forestation and environment awareness programs. Habib et al. [33] incorporated a carbon tax to reduce the carbon emissions and protect the environment. Sepehri et al. [34] provided an investment scheme for carbon reduction and decided the ideal selling cost and replenishment cycle in which carbon would be transmitted due to the ordering and capacity operations; also and carbon cap and trade were controlled. The impact of carbon emissions on an SCM was reduced by Singh et al. [35]. A survey on green mechanisms in the processing and production of food was made by Boye and Arcand [36]; they focused on the topic of environmental sustainability in the agri-food and agriculture sectors and suggested increasing the application of technology to promote greening for food processing and production. Tseng and Lin [37] described the effects of various green designs. The green marketing research topic was reviewed by Wymer and Polonsky [38] who gave an idea on the probability of green trading and its limitations. Shu et al. [39] studied the discharge of carbon in the modeling of SCM. The ways of investment for the green items to the manufacturer was illustrated by Zhang and Zhou [40]. Sana [7] established a two-echelon structural model of an SC with a green level and price-dependent demand. In that model, the retailer and the manufacturer jointly invested in the improvement of the green level. Recently, Liu et al. [41] compared the competition between green and non-green product SCs based on behavior-based pricing for decentralized and centralized cases. However, they ignored the concept of investment to improve the green level of the product. Cost-sharing contracts for this green SC coordination were discussed by Song et al. [42]. In this model, they considered traditional non-green products and green products where the green level of the product was increased; they proved that green products are more profitable for the SC. All of these studies considered a green product or green level, but the investment for green quality improvement and transportation hazards under the conditions of an SSMUID transportation policy has not been studied yet.

In this study, the shortage occurs for the unreliable manufacturer and transportation hazards are solved by applying a transportation hazard cost. A robust distribution method (Mahapatra et al. [43]) was utilized to solve the problems of the study and obtain the maximum profit. Table 1 shows some more works in this field and the nobility of this model.

3. Problems, symbols and presumptions

3.1. Notation

	Index
i	number of hazards ($i = 1, 2, \cdots, s$)
	Decision variables
q	size of initial lot (unit)
ϱ	ascending rate for size of lots ($\varrho > 1$)
n	number of lots (a positive integer)
δ	green level (%)
p	selling price of retailer (\$/unit)
k	safety stock (unit)
P	production rate (unit/day)
L	lead time (days)
	Dependent variable
X	demand of the product, depending on δ, p, n (unit)
Q	ordered quantity, depending on q, ϱ, n (unit)
	Parameters
a	market capacity (unit)

b	scaling parameter for demand
α	fraction $0 < \alpha < 1$ of the ordered quantity produced by the unreliable manufacturer
c_δ	cost per unit of GL δ ($/unit)
C_s	manufacturer's setup cost ($/unit)
C_{mh}	manufacturer's HC ($/unit)
C_{ft}	manufacturer's fixed transportation cost ($/unit)
C_{fc}	manufacturer's fixed carbon emission cost ($/unit)
C_{vt}	manufacturer's variable transportation cost ($/unit)
C_{vc}	manufacturer's variable carbon emission cost ($/unit)
g_0	consumption fuel for return trip from retailer to the manufacture (gallon/mile)
g_1	unit factor for fuel consumption when the vehicle for transportation is filled with goods (gallon/unit/mile)
ω	carbon emission factor for fuel (ton/unit of fuel)
s	number of hazards (a positive integer)
ζ_i	random distance from the manufacturer were the i^{th} hazard occurs (mile)
$E[\zeta_i]$	expected value of the distance from the manufacturer were the i^{th} hazards occurs $i = 1, 2, ..., s$. (mile)
d	distance between retailer and manufacturer (mile)
C_b	hazard cost ($/unit distance of hazard point)
η_1, η_2	scaling parameters for production cost
λ	backorder ratio
π_x	backorder price discount ($/unit)
π_0	marginal profit per unit ($0 \leq \pi_0 \leq \pi_x$) ($ / unit)
r	reorder point (unit)
Y	lead time demand (unit)
C_o	unit cost for ordering for retailer ($/unit)
C_{rh}	retailer's HC ($/unit)
C_δ	development cost for green quality ($/unit)

3.2. Assumptions

1. An SC model with one manufacturer, one retailer and a single item was studied; also the manufacturer was set to be unreliable. The demand $X(\delta, p)$ of the products depends on the green quality δ and selling prices p, and is expressed as $X(\delta, p) = \left(\frac{\delta}{1+\delta}\right) a - bp$, where the market capacity is noted as a and the demand sensitivity according to the sales price is noted by b [7].
2. The manufacturer is unreliable and hides information regarding the quantity and delivery time of the product, which causes a shortage. The manufacturer delivered a percentage αQ of the amount Q ordered by the retailer without any prior information; as a result, the retailer faces a shortage [11]. There is no other manufacturer for that product, so the retailer has to buy the product from that unreliable manufacturer.
3. To save the retailer's HC, the manufacturer delivers the less ordered products in n unequal lots by truck. The sizes of the lots increases in a geometric progression. The size of each lot is a multiple

of the previous lot, that is the 1^{st} lot size is q, the 2^{nd} lot size is ϱq, where $varrho > 1$ and it is a multiple of the 1^{st} lot, the 3^{rd} lot size is $\varrho^2 q$ which is a multiple of the 2^{nd}, ..., the n^{th} lot size is $\varrho^{n-1}q$ which is a multiple of the size of the $n-1^{th}$ lot. This policy is known as the SSMUID policy [5].

4. The number of hazards may increase for the SSMUID policy as the number of transportation jobs increase. The hazardousness not only increases the total cost but also affects the delivery time. The hazard cost depends on the distance traveled by trucks. If d is the total distance between

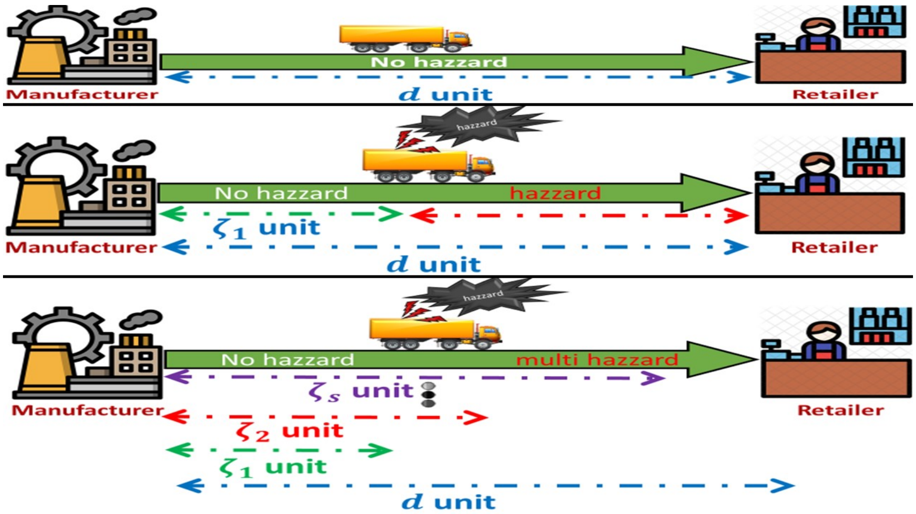

Figure 1. Transportation hazards.

the manufacturer and the retailer and the hazard i occurs at the random distance of ζ_i then the total hazard cost of the manufacturer is $\sum_{i=1}^{s}(d - E[\zeta_i])C_b$, where s is the number of hazardness in one cycle and C_b is the hazard cost (Figure 1). The random distance ζ_i may follow a uniform, triangular, double-triangle, beta, or χ^2 distribution function, and $E[\zeta_i]$ is the expected value of ζ_i.

5. Due to heavy transportation besides an fixed transportation and carbon emission cost (FTCEC), a variable transportation and carbon emission cost (VTCEC) is also considered. The VTCEC depends on the fuel consumption of the trucks [16].

3.3. Problem definition

After getting an order quantity of Q units of products from the retailer, the unreliable manufacturer starts manufacturing. But without informing the retailer, the unreliable manufacturer manufactures $\alpha Q, 0 < \alpha < 1$ quantities of the product which is a fraction of the ordered quantity. Under the conditions of an increasing demand pattern, to reduce the HC of the retailer, the SSMUID transportation policy is applied by the manufacturer. With this policy, all of the manufactured products are delivered in n lots by trucks, and the size of each lot increases by a multiple with the previous lot; that is if the 1^{st} lot size is a q unit, the 2^{nd} lot size is a ϱq unit, the 3^{rd} lot size is a $\varrho^2 q$ unit and the n^{th} lot size is a $\varrho^{n-1}q$ unit. Due to the SSMUID policy, the number of transportation jobs increases, which increases the costs for transportation, carbon emissions and the number of hazards during transportation, which is random.

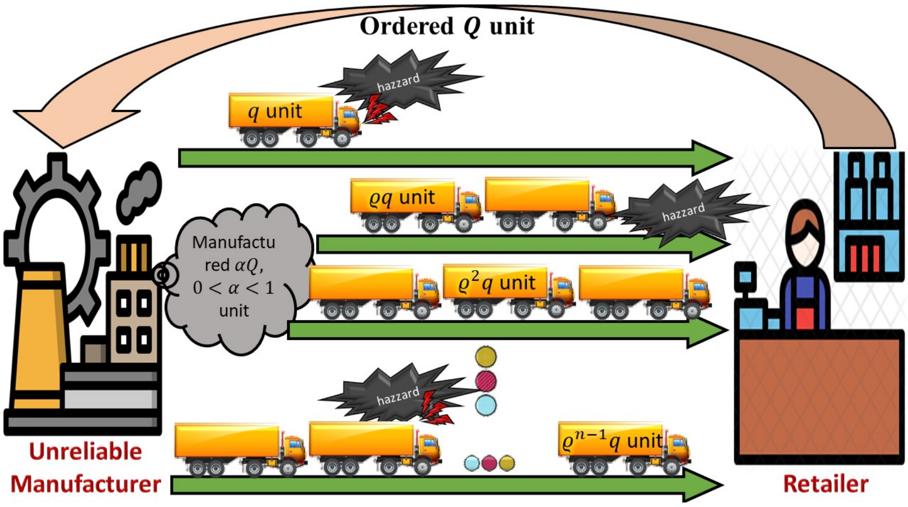

Figure 2. Description of the problem.

Due to the unreliability of the manufacturer and the hazards in transportation, shortages may occur, and the retailer may face problems. The aim of the study was to solve the storage problem and maximize the profit of the SC by applying smart manufacturing, hazard cost and variable backorder price discounts. For environmental issues, a fuel-dependent VTCEC was applied and some investments were incorporated to increase the green level of the product. The demand Y during the LT is a variable (random) with a mean of XL and standard distribution of $\sigma\sqrt{L}$. A distribution-free approach was applied to solve the model. Graphically the problem solved in this study is presented in Figure 2.

4. Model formulation

In this part the mathematical model for different players in the SC is calculated.

4.1. Manufacturer's mathematical model

The number of total produced item is αQ and the sizes of $1^{st}, 2^{nd}, ..., n^{th}$ lots are $q, \varrho q, ..., \varrho^{n-1} q$, respective. Therefore,

$$\alpha Q = q + \varrho q + \varrho^2 q + ... + \varrho^{n-1} q = q\left(\frac{\varrho^n - 1}{\varrho - 1}\right) \tag{4.1}$$

Thus the cycle length $\frac{\alpha Q}{X}$ becomes $\frac{q}{X}\left(\frac{\varrho^n-1}{\varrho-1}\right)$. Figure 3 presents the joint figure for the manufacturer and the retailer. From Figure 3 we can say that the total inventory of the manufacturer is

$$q\alpha Q\left[\frac{1}{P} + \left(\frac{1}{X} - \frac{1}{2P}\right)\left(\frac{\varrho^n - 1}{\varrho - 1}\right) - \frac{1}{2X}\left(\frac{\varrho^n + 1}{\varrho + 1}\right)\right]$$

The average inventory of the manufacturer is

$$q\left[\frac{X}{P} + \left(1 - \frac{X}{2P}\right)\left(\frac{\varrho^n - 1}{\varrho - 1}\right) - \frac{1}{2}\left(\frac{\varrho^n + 1}{\varrho + 1}\right)\right]$$

The costs of the manufacturer are as follows.

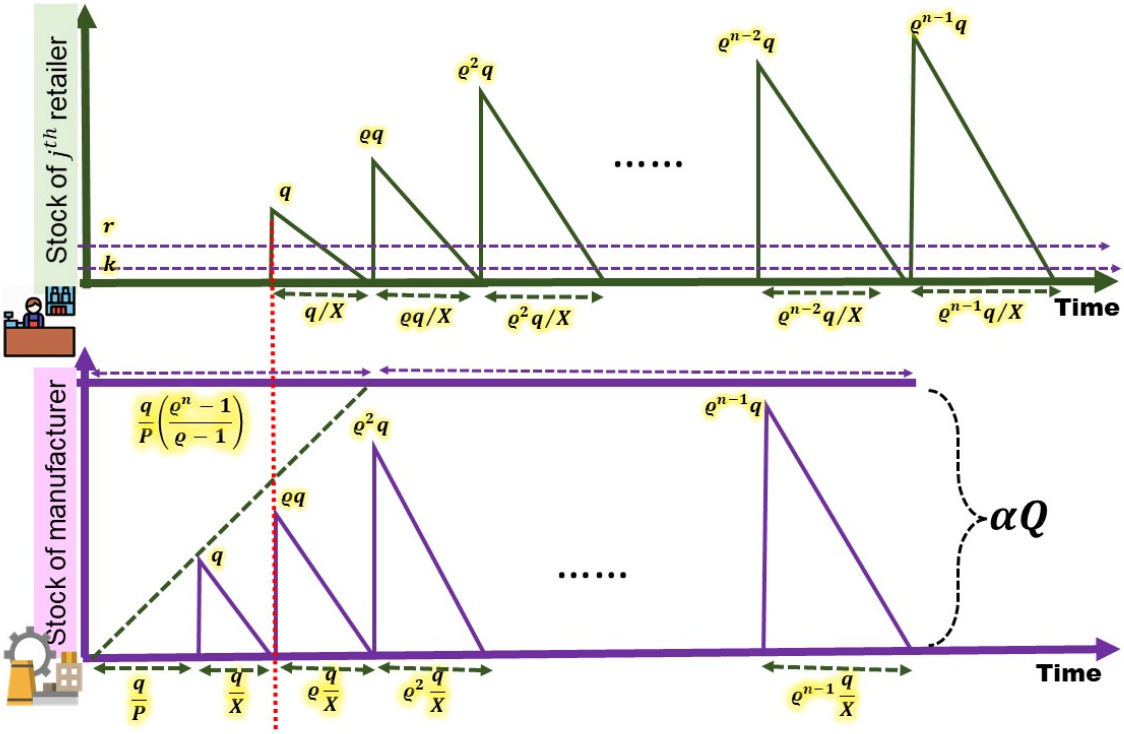

Figure 3. Inventory positions for the manufacturer and retailer.

4.1.1. Manufacturer setup cost

To setup the production process, the manufacturer needs the following cost.

$$\frac{X}{\alpha Q} C_s$$

4.1.2. Manufacturer HC

The cost for holding the inventory for the manufacturer per cycle is as follows:

$$q\left[\frac{X}{P} + \left(1 - \frac{X}{2P}\right)\left(\frac{\varrho^n - 1}{\varrho - 1}\right) - \frac{1}{2}\left(\frac{\varrho^n + 1}{\varrho + 1}\right)\right] C_{mh}$$

4.1.3. Manufacturer transportation and carbon emission cost

The total fixed transportation and carbon emission cost per cycle is

$$nq\frac{X}{\alpha Q}\left(C_{ft} + C_{fc}\right)$$

where C_{fc} and C_{ft} are the fixed carbon emission cost and fixed transportation cost of the manufacturer and the total variable transportation and carbon emission cost is

$$\frac{X}{\alpha Q}\left(2ng_0 + \frac{\varrho^n - 1}{\varrho - 1} q g_1\right)(C_{vt} + \omega C_{vc})$$

where g_0 is the fuel consumed by the empty vehicle upon returning from the retailer, and g_1 is the factor of unit fuel consumed for a loaded vehicle per unit of goods. ω is the carbon emissions factor for the fuel and C_{vc} and C_{vt} are the variable carbon emission cost and variable transportation cost, respectively. Thus one can express the cost related to total carbon emission cost is

$$\frac{X}{\alpha Q}\left[nq\left(C_{ft}+C_{fc}\right)+\frac{X}{\alpha Q}\left(2ng_0+\frac{\varrho^n-1}{\varrho-1}qg_1\right)(C_{vt}+\omega C_{vc})\right]$$

4.1.4. Manufacturer transportation hazard cost

According to Assumption 4, the total cost of transportation hazard for the manufacturer due to transportation is

$$\sum_{i=1}^{s}(d-E[\zeta_i])C_b$$

where the number of hazards is denoted by s and the hazard cost C_b. d is the distance between two players, and ζ_i is the distance of random hazards, which may follow either a triangular, uniform, beta, double-triangle or χ^2 distribution; $E[\zeta_i]$ is the expected value of ζ_i.

4.1.5. Manufacturer production cost

The cost related to production for the manufacturer is

$$\left(\frac{\eta_1}{P}+\eta_2 P\right)X$$

4.1.6. Manufacturer total cost

Therefore, the entire manufacturer's cost is

$$\begin{aligned}TCM\,(q,\varrho,n,\delta,P)&=\frac{X}{\alpha Q}C_s+q\left[\frac{X}{P}+\left(1-\frac{X}{2P}\right)\left(\frac{\varrho^n-1}{\varrho-1}\right)-\frac{1}{2}\left(\frac{\varrho^n+1}{\varrho+1}\right)\right]C_{mh}\\&+\frac{X}{\alpha Q}\left[nq\left(C_{ft}+C_{fc}\right)+\left(2ng_0+\frac{\varrho^n-1}{\varrho-1}qg_1\right)(C_{vt}+\omega C_{vc})\right]+\sum_{i=1}^{s}(d-E[\zeta_i])C_b\\&+\left(\frac{\eta_1}{P}+\eta_2 P\right)X\end{aligned}$$

4.2. Mathematical model of the retailer

Figure 3 shows a figure for the retailer; we can see that the manufacturer sends the order quantity in n lots in each cycle of production with the lot sizes $q,\varrho q,\varrho^2 q,...,\varrho^{n-1}q$. Hence the transportation production batch from the manufacturer to the retailer is

$$q+\varrho q+\varrho^2 q+...+\varrho^{n-1}q=q\left(\frac{\varrho^n-1}{\varrho-1}\right)=\alpha Q$$

and according to Sarkar et al. [15] the production cycle number is

$$\frac{X}{\alpha Q}$$

The costs of the retailer are as follows.

4.2.1. Retailer ordering cost

The cost related to ordering a product for the retailer is given by

$$\frac{X}{\alpha Q} C_o$$

4.2.2. Retailer annual stockout cost

The annual cost for stockout per cycle is as follows

$$\frac{X}{\alpha Q} \left[\pi_x \lambda + \pi_0 (1 - \lambda)\right] E (Y - r)^+$$

where π_0 and λ are the marginal profit and the backorder ratio respectively. $\pi_x(0 \le \pi_x \le \pi_0)$ is the discounted price for the backorder and the expected value of shortage of the retailer is $E (Y - r)^+$.

4.2.3. Holding cost of the retailer

The number of holding items is $\frac{q}{2}\left(\frac{\varrho^n - 1}{\varrho - 1}\right)$ (Figure 3). Also in a cycle the expected backorder is $\lambda E (Y - r)^+$ and the expected lost sales is $(1 - \lambda) E (Y - r)^+$. Thus, the cost for holding the inventory of the retailer is as follows

$$\left[\frac{q}{2}\left(\frac{\varrho^n - 1}{\varrho - 1}\right) + (r - XL) + (1 - \lambda) E (Y - r)^+\right] C_{rh}$$

4.2.4. Total cost of the retailer

Thus, the entire cost for the retailer in a cycle is given by:

$$\begin{aligned} TCR (q, \varrho, n, \delta, \pi_x, A, L) = \quad & \frac{X}{\alpha Q} C_o + \frac{X}{\alpha Q} \left[\pi_x \lambda + \pi_0 (1 - \lambda)\right] E (Y - r)^+ \\ & + \left[\frac{q}{2}\left(\frac{\varrho^n - 1}{\varrho - 1}\right) + (r - XL) + (1 - \lambda) E (Y - r)^+\right] C_{rh} \end{aligned}$$

4.3. Joint total profit of the SC

The investment for green quality development by the retailer and the manufacturer in total is δC_δ. Therefore, in a cycle the joint profit of the SC is as follows:

$$\begin{aligned} & JTP (q, \varrho, n, \delta, \pi_x, p, P, k, L) \\ & = \left(p - \frac{\eta_1}{P} - \eta_2 P\right)\left\{\left(\frac{\delta}{1 + \delta}\right) a - bp\right\} - \left[\frac{\left(\frac{\delta}{1+\delta}\right) a - bp}{\alpha Q} C_s \right. \\ & + q \left\{\frac{\left(\frac{\delta}{1+\delta}\right) a - bp}{P} + \left(1 - \frac{\left(\frac{\delta}{1+\delta}\right) a - bp}{2P}\right)\left(\frac{\varrho^n - 1}{\varrho - 1}\right) - \frac{1}{2}\left(\frac{\varrho^n + 1}{\varrho + 1}\right)\right\} C_{mh} \\ & + \frac{\left(\frac{\delta}{1+\delta}\right) a - bp}{\alpha Q}\left[nq\left(C_{ft} + C_{fc}\right) + \left(2ng_0 + \frac{\varrho^n - 1}{\varrho - 1} qg_1\right)(C_{vt} + \omega C_{vc})\right] + \sum_{i=1}^{s} (d - E[\zeta_i]) C_b \end{aligned}$$

$$+\frac{\left(\frac{\delta}{1+\delta}\right)a-bp}{\alpha Q}C_o+\frac{\left(\frac{\delta}{1+\delta}\right)a-bp}{\alpha Q}\left\{\pi_x\lambda+\pi_0\left(1-\lambda\right)\right\}\frac{1}{2}\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)$$
$$+\left\{\frac{q}{2}\left(\frac{\varrho^n-1}{\varrho-1}\right)+k\sigma\sqrt{L}+(1-\lambda)\frac{1}{2}\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)\right\}C_{rh}+\delta C_\delta\Bigg]$$

5. Solution methodology

This section provides the solution process and the corresponding optimal solutions.

5.1. Distribution-free approach

According to Scarf [44], the following inequality holds for all $F\in\Omega$

$$E\left(Y-r\right)^+=\frac{1}{2}\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)$$

Therefore, the problem is reduced to

$$\max_{F\in\Omega} JTP\left(p,\delta,q,\varrho,n,P,k,L\right)$$

5.2. Methodology

The demand function is $X(\delta,p)=\left(\frac{\delta}{1+\delta}\right)a-bp$. $a(\geq 0)$ is the capacity of the market, $b(>0)$ is a parameter for price sensitivity. Here, $\frac{\partial X}{\partial\delta}=a\left(1+\delta\right)^{-2}$ for all δ and as $b(>0)$, $\frac{\partial X}{\partial p}=-b<0$. Thus it is clear that demand is directly proportional to the level of greenness and inversely proportional to the product's price. The profit function can be written as

$$J=-\frac{X}{\alpha Q}R_1-\alpha QR_2+XR_3-R_4$$

where the values of R_1,R_2,R_3 and R_4 are given in Appendix B. Denoting the joint profit function JTP by simply J and differentiating it J partially with respect to L two times one can get

$$\frac{\partial^2 J}{\partial L^2}=\frac{X}{\alpha Q}\frac{\left(\sqrt{k^2+1}-k\right)\sigma\left\{(\pi_0(1-\lambda)+\lambda\pi_x\right\}}{8L^{3/2}}+\frac{(1-\lambda)\left(\sqrt{k^2+1}-k\right)\sigma}{8L^{3/2}}+\frac{k\sigma}{4L^{3/2}}>0$$

Thus, for a fixed q,ϱ,n,δ,p,P,k and L the profit function (J) is concave with respect to L. Thus, the maximum profit exists within the interval $[L_i,L_{i-1}]$. Again, for $L\in[L_i,L_{i-1}]$, partially differentiating J with respect to $p,\delta,q,\varrho,P$and k gives

$$\frac{\partial J}{\partial p}=b\left(\frac{R_1}{\alpha Q}-\frac{\alpha Q}{2P}C_{mh}-R_3\right)+a\left(\frac{\delta}{1+\delta}\right)-bp \tag{5.1}$$
$$\frac{\partial J}{\partial\delta}=-\frac{a}{(\delta+1)^2}\left(\frac{R_1}{\alpha Q}-\frac{\alpha Q}{2P}C_{mh}-R_3\right)-C_\delta \tag{5.2}$$
$$\frac{\partial J}{\partial q}=\frac{X}{\alpha Q}\left\{\frac{R_1}{q}-n\left(C_{ft}+C_{fc}\right)\right\}-\frac{\alpha Q}{q}R_2-\frac{X}{P}C_{mh} \tag{5.3}$$

$$\frac{\partial J}{\partial \varrho} = \varrho_1 \left(\frac{X}{\alpha Q} R_1 - \alpha Q R_2 \right) + \frac{1}{2} \left(\frac{\varrho^n + 1}{\varrho + 1} \right) \left(\frac{n \varrho^{n-1}}{\varrho^n + 1} - \frac{1}{\varrho + 1} \right) C_{mh} \tag{5.4}$$

$$\frac{\partial J}{\partial P} = X \left\{ \frac{1}{P^2} \left(\eta_1 - \frac{1}{2} \alpha Q C_{mh} - q C_{hm} \right) - \eta_2 \right\} \tag{5.5}$$

$$\frac{\partial J}{\partial k} = \left[-\frac{X}{\alpha Q} \frac{1}{2} \{ \pi_x \lambda + \pi_0 (1 - \lambda) \} \left(\frac{k}{\sqrt{1 + k^2}} - 1 \right) - \left\{ 1 + \frac{1}{2} (1 - \lambda) \left(\frac{k}{\sqrt{1 + k^2}} - 1 \right) \right\} C_{rh} \right] \sigma \sqrt{L} \tag{5.6}$$

By equating these derivatives to 0, the optimal values are obtained as follows:

$$p^* = \frac{R_1}{\alpha Q} - \frac{\alpha Q}{2P} C_{mh} - R_3 + \frac{a}{b} \left(\frac{\delta}{1 + \delta} \right)$$

$$\delta^* = \sqrt{\frac{a}{c\delta} \left(R_3 + \frac{\alpha Q}{2P} C_{mh} - \frac{R_1}{\alpha Q} \right)} - 1$$

$$q^* = \frac{P}{X} \left[\frac{X}{\alpha Q} \left\{ R_1 - nq \left(C_{ft} + C_{fc} \right) \right\} - \alpha Q R_2 \right] C_{mh}$$

$$\varrho^* = -1 - \frac{(\varrho^n + 1) \left(\frac{n\varrho^{n-1}}{\varrho^n + 1} - \frac{1}{\varrho + 1} \right) C_{mh}}{2 \varrho_1 \left(\frac{X}{\alpha Q} R_1 - \alpha Q R_2 \right)}$$

$$P^* = \sqrt{\frac{\eta_1 - \frac{1}{2} \alpha Q C_{mh} - q C_{hm}}{X \eta_2}}$$

$$k^* = \sqrt{1 + k^2} \left[1 - \frac{2 C_{rh}}{\frac{X}{\alpha Q} \Pi + (1 - \lambda) C_{rh}} \right]$$

Proposition 5.1. ***For any*** $L \in [L_i, L_{i-1}]$***, the Hessian matrix for*** J ***is negative definite at the point*** $(p^*, \delta^*, q^*, \varrho^*, P^*, k^*)$***; this means that the value of the profit function is maximum (globally) at*** $(p^*, \delta^*, q^*, \varrho^*, P^*, k^*)$

Proof. Follow Appendix D. □

6. Numerical experiments

A numerical experiment was performed to establish the reality of the study. The optimality conditions and maximum profit are provided in Table 3. The concavity with respect to different decision variables are presented in Figures 4 – 6. Hota et al. [5] and Sana [7] provideed the parametric values as follows: C_S = \$1500; C_{hm} = 0.02 (\$/unit); C_{ft} = 0.3 (\$/unit); C_{fc} = 0.2 (\$/unit); C_{vt} = 0.2 (\$/unit); C_{vc} = 0.1 (\$/unit); g_0 = 20 (gallon); g_1 = 0.05 (gallon/unit); ω = 0.01015; π_x = 78.23 (\$/unit); π_0 = 150 (\$/unit); λ = 0.7; σ = 7; d = 115 (mile); s = 5; ζ_1 = 57 (mile); ζ_2 = 32 (mile); ζ_3 = 93 (mile); C_b = 2.5 (\$/distance); η_1 = 105; η_2 = 0.0006; C_O = 40 (\$/unit); C_{rh} = 1.01 (\$/unit); C_δ = 250 (\$/unit); a = 850; b = 5. Then by using Wolfram Mathematica, one can obtain the optimized value for the decision variables and profit of the entire system.

Table 3 establishes that the maximum profit can be obtained if ζ_i follows a uniform distribution. Also the values of the dependent variables, that is the order quantity, number of delivered items and

Table 2. Value of scaling parameters for different distributions.

Uniform distribution	Triangular distribution	Double-triangular distribution	Beta distribution	χ^2 distribution
(a_i, b_i)	(a_i, b_i, c_i)	(a_i, b_i, c_i)	(α_i, β_i)	(κ_i)
$(0.03, 0.07)$	$(0.03, 0.04, 0.07)$	$(0.03, 0.04, 0.07)$	$(0.03, 0.07)$	0.03
$(0.035, 0.07)$	$(0.035, 0.045, 0.07)$	$(0.035, 0.045, 0.07)$	$(0.035, 0.07)$	0.035
$(0.04, 0.08)$	$(0.04, 0.045, 0.08)$	$(0.04, 0.045, 0.08)$	$(0.04, 0.08)$	0.04
$(0.04, 0.06)$	$(0.04, 0.04, 0.07)$	$(0.04, 0.04, 0.08)$	$(0.04, 0.07)$	0.045
$(0.03, 0.075)$	$(0.045, 0.04, 0.07)$	$(0.045, 0.04, 0.08)$	$(0.03, 0.075)$	0.03

Table 3. Optimality table.

Distribution of ζ_i	p^*(\$/unit), δ^*(percent), q^* (unit) ϱ^*, n^*, L^* (days), P^* (unit/day), k^*(unit)	Maximum profit JTP
Uniform distribution	81.03, 15.34, 356.73, 1.05, 3, 3, 410.06, 2.82	\$25632.7
Triangular distribution	81.03, 15.46, 356.73, 1.05, 3, 3,410.45, 2.8	\$25603.5
Double triangular distribution	82.1, 15.7, 355.91, 1.05, 3, 3, 411.12, 2.79	\$25558.3
Beta distribution	81.21, 14.94, 354.85, 1.05, 3, 3, 411.1, 2.9	\$24974.2
χ^2 distribution	81.17, 14.88, 356.1, 1.05, 3, 3, 409.16, 2.62	\$25420.2

Table 4. Values of the dependent variables.

Dependent variable	Value
Order quantity (Q)	1300 unit
Percentage of the order quantity produced by the manufacturer (α)	86.5%
Number of items deviled to the retailer by the manufacturer (αQ)	1124.6 unit
Percentage of the unreliability of the manufacturer	13.5%
Demand for the product (X)	392.83 unit

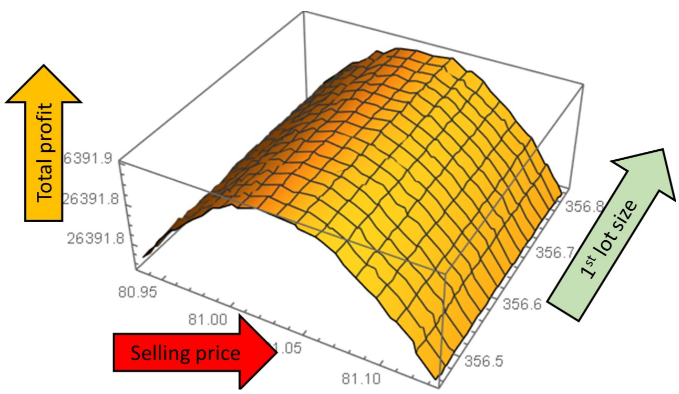

Figure 4. Concavity with respect to 1st lot size and selling price.

demand for the product are given in Table 4.

The Hessian matrix at the optimal point is

$$H = \begin{bmatrix} -10 & 3.18 & -0.02 & -9.77 & -0.15 & 0 \\ 3.18 & -30.58 & 0.01 & 6.22 & 0 & 0.1 \\ -0.02 & 0.01 & -0.01 & -3.2 & 0 & -0.03 \\ -9.77 & 6.22 & -3.2 & -1362.32 & -0.03 & -11.97 \\ -0.15 & 0 & 0 & -0.03 & -0.11 & 0 \\ 0 & 0.1 & -0.03 & -11.97 & 0 & -7.97 \end{bmatrix}$$

The eigenvalues of the Hessian matrix are −1362.53, −31.0169, −9.47099, −7.8589, −0.107637 and −0.00248161 which are all negative. Therefore, the Hessian matrix is negative definite, which shows that $(p^*, \delta^*, q^*, \varrho^*, P^*, k^*)$ =(81.03, 15.34, 356.73, 1.05, 410.06, 2.82) is a point of global maximum for the profit function.

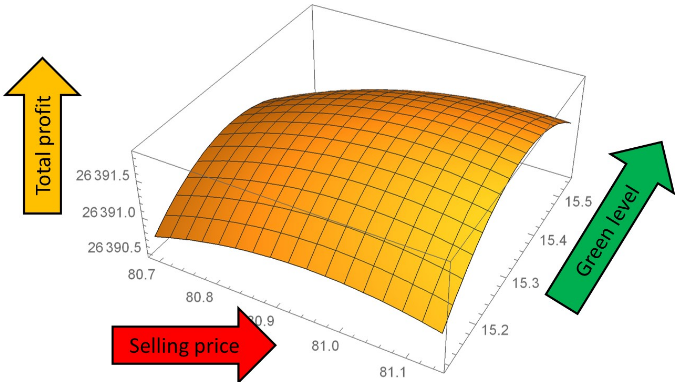

Figure 5. Concavity with respect to GL and selling price.

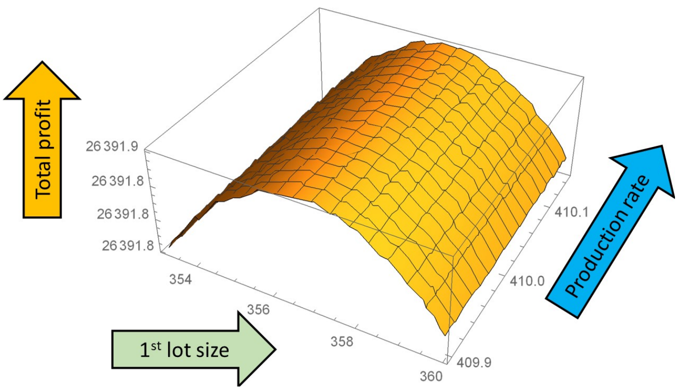

Figure 6. Concavity with respect to production rate and size of 1st lot.

6.1. Comparative study

In this study, the manufacturer utilizes the SSMUID policy. The other popular policies are the SSSD policy and the SSMD policy. In the SSSD policy, the manufacturer transports all of the manufactured items in one lot. In the SSMD policy, the manufacturer transports the produced items in different lots with equal lot sizes. The maximum profits for the applications of different shipment strategies are shown in detail in Table 5. From Table 5, one can say that the SSMUID transportation policy is more beneficial than the SSSD and SSMD transportation policies. The SSMUID policy results in approximately \$159.9 and \$89.5 more profit than the SSSD policy and the SSMD policy respectively.

Table 5. Optimality table.

Shipment policy	Number of lots	Lot size	Total profit
SSSD policy	1	1124.6 unit	\$25472.8
SSMD policy	3	374.87 unit per lot	\$25543.2
SSMUID policy (this model)	3	1st lot - 356.73 unit 2nd lot - 374.57 unit 3rd lot - 393.29 unit	\$25632.7

From Table 5 it can be established that the application of the SSMUID policy generates more profit than the SSSD policy and SSMD policy under the conditions of this study. It is also shown that the SSMD policy is more beneficial than the SSSD policy under the conditions of this study.

6.2. Case study

To validate this model, a case study was establish. For that purpose, a survey of a medicine company in West Bengal was conducted, and various data about the parameters was collected. The collected values for the parameters were as follows: $C_S = \$1500$; $C_{hm} = 0.02$ (\$/unit); $C_{ft} = 0.35$ (\$/unit); $C_{fc} = 0.25$ (\$/unit); $C_{vt} = 0.25$ (\$/unit); $C_{vc} = 0.15$ (\$/unit); $g_0 = 25$ (gallon); $g_1 = 0.1$ (gallon/unit); $\omega = 0.0105$; $\pi_x = 78.23$ (\$/unit); $\pi_0 = 150$ (\$/unit); $\lambda = 0.7$; $\sigma = 7$; $d = 115$ (mile); $s = 5$; $\zeta_1 = 57$ (mile); $\zeta_2 = 32$ (mile); $\zeta_3 = 93$ (mile); $C_b = 2.5$ (\$/distance); $\eta_1 = 105$; $\eta_2 = 0.0006$; $C_O = 40$ (\$/unit); $C_{rh} = 1.01$ (\$/unit); $C_\delta = 250$ (\$/unit); $a = 850$; $b = 5$. From the data, it is observed that ζ_i follows a uniform distribution, and that the profit of the company was obtained as \$25075.43.

This fact established that the medicine company will benefit by nearly \$557.27, this is why the medicine company has agreed to adopt this policy of this study.

6.3. Sensitivity study

Table 6 describes the effects on the joint total profit as a result of changing some parameters from −50% to +50%. The other parameters either had no effect on the total profit or had a very small effect on the total profit.

From Table 6, the following points can be concluded

- The total profit diminishes with an increase in cost and increases with a decrease in cost; that is a natural fact. This establishes the reliability of the study.
- The greening cost C_δ was the most sensitive parameter in the study, and the industry managers should maintain focus on it.

Table 6. Sensitivity table.

Parameter	Change	Change in JTP (%)	Parameter	Change	Change in JTP (%)
C_S	−50%	+1.32	C_{mh}	−50%	+0.03
	−25%	+0.62		−25%	+0.01
	+25%	−0.51		+25%	−0.01
	+50%	−0.92		+50%	−0.03
C_{ft}	−50%	+0.53	C_{fc}	−50%	+0.28
	−25%	+0.18		−25%	+0.08
	+25%	−0.19		+25%	−0.08
	+50%	−0.52		+50%	−0.27
C_{rh}	−50%	+1.27	C_{δ}	−50%	+8.52
	−25%	+0.67		−25%	+3.92
	+25%	−0.69		+25%	−3.81
	+50%	−1.25		+50%	−7.96

- The HC of the manufacturer C_{mh} was the least sensitive parameter in the study. This was related to the location of the manufacturer. Since the manufacturer is located in a rural area, the HC of the manufacturer is low.
- Although the retailer's HC was the 2nd most sensitive parameter in the study, it had minimal effect on the profit.
- Other parameters, i.e., C_s, C_{ft} and C_{fc}, also had a smaller effect on the profit. Among the parameters, the setup cost C_S had the greatest effect on the entire system profit.

The effects of the aforementioned parameters are graphically represented in Figure 7.

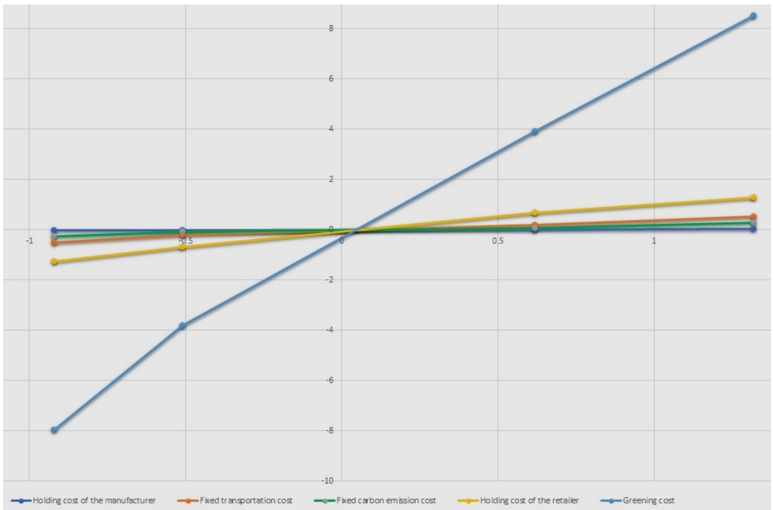

Figure 7. Effect on profit as a result of changing different parameters.

6.4. Discussion

From Table 3 one can conclude that the maximum profit obtained when ζ_i follows a uniform distribution; the maximum profit was determined to be \$26391.9. The optimal value of the selling price is 81.03 (\$/unit), the optimal value for the green level is 15.34, the 1st lot size was determined to be 356.73 (unit) and the increasing rate of lot size should be 1.05. The number of shipments should be three. The concavity of the profit function with respect to decision variables is graphically shown in Figure 4, Figure 5 and Figure 6.

The SSMUID transportation policy is more beneficial than the other transportation mode which is shown in Table 4; a comparative study was conducted with the existing literature.

To validate the present model, a case study with real data was conducted the data were collected from a medicine company located in West Bengal. Due to the use of the SSMUID policy, the green level of the product and investment in transportation hazards can be really helpful for any industry, which has been demonstrated by this case study. This is why the medicine company happily adopted the concept of this study. Due to a company's policy, the name and the other details of the company are not provided here.

From the sensitivity described in Table 6 it can be understood that the green cost and HC of the manufacturer were the highest and least sensitive parameters of the study, respectively.

A fixed ordering cost and setup cost are two limitations of this study. The research can be extended by introducing intelligent technologies [17]. Investments can reduce setup cost and ordering cost [45]. By improving the quality of the production process one can control the shortages problem [46]. For any SC, the lead time plays a very important role. Thus one can extend the current study by using the concept of controllable lead time [4].

7. Managerial insights

To evaluate the business start and progress, there needs to be statistical data and scientific observations. Industry managers are subject to more benefits through the implications of moderate research ideas and technologies.

A variable rate of production is a crucial issue for any industry. Here, the considerable unit production cost is a function of the rate of production. The variable production rate controls the fluctuating market demand, survives market competition, prevents shortages, increases the good reputation, confirms to customers the availability of the products and yields a desirable profit. These ideas were demonstrated via this proposed research. Hence industrial managers should focus on it.

Another important matter is the SSMUID policy. Although the number of shipments increases for this policy, the HC reduces significantly. However, for the unreliable manufacturer and the reliable retailer, the SSMUID policy is very much helpful to the retailer as a smooth-running business strategy. Moreover, it was also proven that the SSMUID policy is much better than the SSSD policy and SSMD policy. Hence, the marketing manager should accept this policy.

Transportation hazards are a gigantic problem for each and every manufacturing base marketing system. In this study, probable transportation disruption and arrangements for overcome it is described elaborately. Hence, alternative arrangements and advance investments should be made by the industry manager to avoid any type of transportation disruption situation.

Another vital and widely studied matter is carbon emissions, which seriously harms the environment. Due to increase in delivery through transportation, production, etc., there arise more carbon emissions. In this study, the carbon emission cost with green level products makes the model more environmentally friendly. By applying this type of model, the industry manager can produce eco-friendly products and control carbon emissions, which makes products more acceptable for the conscientious customers.

In this study, other improvements were related to safety stock, minimizing the LT and controlling the stockout situation. Safety stock makes the marketing strategy more reliable to the customers, whereas an optimum LT helps to meet the customer's need within said time; additionally, the strategy for controlling the stockout situation helps to survive shortages and promote an overall good reputation. An industry manager who applies all of these ideas can make more profit.

8. Conclusions

A two-echelon SC with an unreliable manufacturer, reliable retailer and a single sort of item was studied. The demand for the product varies with the price and green level of the product. The retailer faces a shortage problem due to the unreliability of the manufacturer and transportation hazards. That influences the notoriety of the retailer as well as the notoriety of the company. To reduce the lost sales, the retailer offers a backorder price discount. The unreliable manufacturer should apply a modern transportation methodology to solve the shortage issue and diminish transportation hazards. Finally, the storage problem is solved by the successful application of the shipment strategy and investment for the hazard problem. To demonstrate the practicality of the study, a numerical example with various case studies and graphical representation has been given. From the numerical experiment, one can find that the system profit is maximized when the random variable corresponding to the distance of hazards follows a uniform distribution. One can conclude that the SSMUID policy is much more beneficial for optimizing the total system profit compared to the other transportation policies. To improve environmental sustainability, the manufacturer and the retailer should both invest in the improvement of green level. This research can be extended by enabling the role of blockchain technology in supply chain management [47–49].

Appendix

A. Abbreviations

LT - "lead time"; TH - "Transportation hazard"; TC - "Transportation cost"; FTC - "Fixed transportation cost"; VTC - "Variable transportation cost"; CEC - "Carbon emission cost"; FCEC - "Fixed carbon emission cost"; VCEC - "Variable carbon emission cost"; FTCEC - "Fixed transportation and carbon emission cost"; VTCEC - "Variable transportation and carbon emission cost"; GL - "Green level"; PR - "Production rate"; DDO - "Demand depends on"; CEI - "carbon emission index"; SP - "Selling price"; SC - "Supply chain"; SCM - "Supply chain management"; SSSD - "Single-setup-single-delivery"; SSMD - "Single-setup-multi-delivery"; SSMUD - "Single-setup-multi-unequal-delivery"; SSMUID - "Single-setup-multi-unequal-increasing-delivery"

B. Values of R_1, R_2, R_3, R_4

$$R_1 = (C_S + C_O) + nq\left(C_{ft} + C_{fc}\right) + 2ng_0\left(C_{vt} + \omega C_{vc}\right) + \frac{1}{2}\sigma\sqrt{L}\left\{\pi_x\lambda + \pi_0(1-\lambda)\left(\sqrt{1+k^2} - k\right)\right\},$$

$$R_2 = \left(1 - \frac{X}{2P}\right)C_{mh} - \frac{1}{2}C_{rh}, R_3 = \left(p - \frac{\eta_1}{P} - \eta_2 P\right) - \frac{q}{P}C_{mh} - g_1\left(C_{vt} + \omega C_{vc}\right),$$

$$R_4 = \sum_{i=1}^{s}(d - E[\zeta_i])C_b + \left\{1 + \frac{1}{2}(1-\lambda)\left(\sqrt{1+k^2} - k\right)\right\}\sigma\sqrt{L}C_{rh} - \frac{1}{2}\frac{\varrho^n + 1}{\varrho + 1}C_{mh} + \delta C_\delta,$$

$$R_5 = \left(\frac{\alpha Q}{X} + \frac{R_2}{q} + \frac{1}{P}C_{mh}\right), \varrho_1 = \left(\frac{n\varrho^{n-1}}{\varrho^n - 1} - \frac{1}{\varrho - 1}\right), \Pi = \left\{\pi_x\lambda + \pi_0\left(1 - \lambda\right)\right\},$$

$$K_1 = \left(\frac{k}{\sqrt{1+k^2}} - 1\right), R_6 = \frac{\alpha Q}{q}\left(\frac{R_2}{X} - \frac{1}{2P}C_{mh}\right), R_7 = \left(\frac{\alpha Q}{2P}C_{mh} - \frac{1}{\alpha Q}R_1\right),$$

$$R_8 = \left\{R_1 + nq\left(C_{ft} + C_{fc}\right)\right\}, R_9 = \left(2\frac{\alpha Q}{q}R_2 + \frac{X}{P}C_{hm}\right),$$

$$\varrho_2 = \left\{\frac{n(n-1)\varrho^{n-2}\left(\varrho^n - 1\right) - n^2\varrho^{2n-2}}{\left(\varrho^n - 1\right)^2} + \frac{1}{(\varrho - 1)^2}\right\}\left(\frac{X}{\alpha Q}R_1 - \alpha Q R_2\right),$$

$$-\left(\frac{X}{\alpha Q}R_1 + \alpha Q R_2\right)\varrho_1^2 + \frac{1}{2}\left(\frac{\varrho^n + 1}{\varrho + 1}\right)^2\left(\frac{n\varrho^{n-1}}{\varrho^n + 1} - \frac{1}{\varrho + 1}\right)^2 C_{mh}$$

$$+\frac{1}{2}\left(\frac{\varrho^n + 1}{\varrho + 1}\right)\left\{\frac{n(n-1)\varrho^{n-1}\left(\varrho^n + 1\right) - n^2\varrho^{2n-2}}{\left(\varrho^n + 1\right)^2} + \frac{1}{(\varrho - 1)^2}\right\}C_{mh},$$

$$R_{10} = \frac{X}{P^2}\left(1 - \frac{\alpha Q}{2}\right)C_{hm}, K_2 = \left\{\frac{1}{\sqrt{1+k^2}} - \frac{k^2}{(1+k^2)^{\frac{3}{2}}}\right\}$$

C. 1st-order and 2nd-order partial derivatives of the profit function

$$\frac{\partial X}{\partial p} = -b, \frac{\partial X}{\partial \delta} = \frac{a}{(1+\delta)^2}, \frac{\partial}{\partial q}(\alpha Q) = \frac{\alpha Q}{q}, \frac{\partial}{\partial \varrho}(\alpha Q) = \alpha Q\varrho_1 J_{pp} = -b,$$

$$J_{\delta p} = \frac{a}{(1+\delta)^2}, J_{qp} = bR_5, J_{\varrho p} = \frac{b}{\alpha Q}\varrho_1 R_1, J_{Pp} = (1 - bX)\eta_2,$$

$$J_{kp} = \frac{b}{2\alpha Q}\sigma\sqrt{L}\Pi K_1, J_{p\delta} = \frac{a}{(\delta+1)^2}, J_{\delta\delta} = \frac{2C_\delta}{1+\delta}, J_{q\delta} = \frac{a}{(1+\delta)^2}R_6,$$

$$J_{\varrho\delta} = \frac{a}{(1+\delta)^2}\varrho_1 R_7, J_{P\delta} = \frac{a}{(1+\delta)^2}\frac{\eta_2}{X}, J_{k\delta} = -\frac{a}{(1+\delta)^2}\frac{1}{2}\sigma\sqrt{L}\Pi K_1, J_{pq} = bR_5,$$

$$J_{\delta q} = \frac{a}{(1+\delta)^2}\frac{\alpha Q}{q}R_6, J_{qq} = -2\frac{X}{\alpha Q}\frac{1}{q^2}R_8, J_{\varrho q} = -\varrho_1 R_9, J_{Pq} = R_{10},$$

$$J_{kq} = \frac{1}{2q^2}\frac{X}{\alpha Q}\sigma\sqrt{L}\Pi K_1, J_{p\varrho} = \frac{b}{\alpha Q}\varrho_1 R_1, J_{\delta\varrho} = \frac{a}{(1+\delta)^2}\varrho_1 R_7, J_{q\varrho} = -\varrho_1 R_9,$$

$$J_{\varrho\varrho}=\varrho_2, J_{P\varrho}=-\varrho_1\frac{X}{2P^2}\alpha QC_{hm}, J_{k\varrho}=\varrho_1\frac{1}{2}\frac{X}{\alpha Q}\sigma\sqrt{L}\Pi K_1, J_{pP}=(1-bX)\,\eta_2,$$
$$J_{\delta P}=\frac{a}{(1+\delta)^2}\frac{\eta_2}{X}, J_{qP}=R_{10}, J_{\varrho P}=-\varrho_1\frac{X}{2P^2}\alpha QC_{hm}, J_{PP}=-\frac{2\eta_2}{P}, J_{kP}=0,$$
$$J_{pk}=\frac{b}{2\alpha Q}\sigma\sqrt{L}\Pi K_1, J_{\delta k}=-\frac{a}{(1+\delta)^2}\frac{1}{2}\sigma\sqrt{L}\Pi K_1, J_{qk}=\frac{1}{2q^2}\frac{X}{\alpha Q}\sigma\sqrt{L}\Pi K_1,$$
$$J_{\varrho k}=\varrho_1\frac{1}{2}\frac{X}{\alpha Q}\sigma\sqrt{L}\Pi K_1, J_{Pk}=0, J_{kk}=-\frac{1}{2}\left[\frac{X}{\alpha Q}\sigma\sqrt{L}\Pi-(1-\lambda)\right]K_2$$

D. Proof of Proposition 5.1

For any value of $L\in[L_i,L_{i-1}]$, the principal minors of the Hessian matrix for the profit function JTP at the point $(p^*,\delta^*,q^*,\varrho^*,P^*,k^*)$ is given as follows. The first-order minor is $|H_{11}|=-\frac{1}{2}K_2\left(\frac{X\sqrt{L}\Pi\sigma}{\alpha Q}-(1-\lambda)\right)<0$
The second-order minor is

$$|H_{22}|=\frac{X\eta_2K_2\sqrt{L}\Pi\sigma}{\alpha \mathrm{Q}P}+\frac{\lambda\eta_2K_2}{P}-\frac{\eta_2K_2}{P}$$

Clearly, $|H_{22}|>0$ if $\frac{X\eta_2K_2\sqrt{L}\Pi\sigma}{\alpha \mathrm{Q}P}+\frac{\lambda\eta_2K_2}{P}>\frac{\eta_2K_2}{P}$.
The third-order minor is

$$|H_{33}|=\frac{X^2\eta_2\varrho_1^2L\Pi\mathrm{K}_1^2\sigma^2}{2\alpha\mathrm{Q}^2P}-\frac{1}{2}K_2\left(\lambda+\frac{X\sqrt{L}\Pi\sigma}{\alpha\mathrm{Q}}-1\right)\left(\frac{\alpha\mathrm{Q}^2X^2\varrho_1^2C_{\mathrm{hm}}^2}{4P^4}-\frac{2\eta_2\varrho_2}{P}\right).$$

which is negative if $\frac{X^2\eta_2\varrho_1^2L\Pi\mathrm{K}_1^2\sigma^2}{2\alpha\mathrm{Q}^2P}<\frac{1}{2}K_2\left(\lambda+\frac{X\sqrt{L}\Pi\sigma}{\alpha\mathrm{Q}}-1\right)\left(\frac{\alpha\mathrm{Q}^2X^2\varrho_1^2C_{\mathrm{hm}}^2}{4P^4}-\frac{2\eta_2\varrho_2}{P}\right)$.
The fourth order minor is

$$|H_{44}|=\Omega_1+\Omega_2+\Omega_3+\Omega_4>0$$

where

$$\begin{aligned}
\Omega_1 &= \frac{X^4K_2\varrho_1^2\sqrt{L}\Pi R_8\sigma C_{\mathrm{hm}}^2}{4P^4q^2}+\frac{\alpha\mathrm{Q}\lambda X^3K_2\varrho_1^2R_8C_{\mathrm{hm}}^2}{4P^4q^2}+\frac{\alpha\mathrm{Q}X^3K_2\varrho_1^2R_8C_{\mathrm{hm}}^2}{4P^4q^2}\\
&\quad-\frac{X^4\varrho_1^2L\Pi\mathrm{K}_1^2\sigma^2C_{\mathrm{hm}}^2}{16P^4q^4}>0\\
\Omega_2 &= -\frac{2X^2\eta_2K_2\varrho_2\sqrt{L}\Pi R_8\sigma}{\alpha\mathrm{Q}^2Pq^2}+\frac{X^2\eta_2\varrho_2L\Pi\mathrm{K}_1^2\sigma^2}{2\alpha\mathrm{Q}^2Pq^4}-\frac{X^3\eta_2\varrho_1^2L\Pi\mathrm{K}_1^2R_8\sigma^2}{\alpha\mathrm{Q}^3Pq^2}\\
&\quad+\frac{X^2\eta_2\varrho_1^2L\Pi\mathrm{K}_1^2R_9\sigma^2}{\alpha\mathrm{Q}^2Pq^2}>0\\
\Omega_3 &= +\frac{X^2\varrho_1^2L\Pi\mathrm{K}_1^2R_{10}^2\sigma^2}{4\alpha\mathrm{Q}^2}-\frac{X\eta_2K_2\varrho_1^2\sqrt{L}\Pi R_9^2\sigma}{\alpha\mathrm{Q}P}+\frac{XK_2\varrho_2\sqrt{L}\Pi R_{10}^2\sigma}{2\alpha\mathrm{Q}}\\
&\quad-\frac{2\lambda X\eta_2K_2\varrho_2R_8}{\alpha\mathrm{Q}Pq^2}>0
\end{aligned}$$

$$\Omega_4 = \frac{2X\eta_2K_2\varrho_2R_8}{\alpha QPq^2} - \frac{\lambda\eta_2K_2\varrho_1^2R_9^2}{P} + \frac{\eta_2K_2\varrho_1^2R_9^2}{P} + \frac{1}{2}\lambda K_2\varrho_2R_{10}^2 - \frac{1}{2}K_2\varrho_2R_{10}^2 > 0$$

The fifth-order minor is

$$H_{55} = \frac{2C_\delta}{\delta+1}H_{44} - \frac{aR_6}{(\delta+1)^2}\Omega_5 - \frac{a\varrho_1R_7}{(\delta+1)^2}\Omega_6 - \frac{a\eta_2}{(\delta+1)^2X}\Omega_7 - \frac{a\sqrt{L}\Pi\mathrm{K}_1\sigma}{2(\delta+1)^2}\Omega_8.$$

which is less than 0 for $\Omega_5, \Omega_6, \Omega_7, \Omega_8$ with $\frac{aR_6}{(\delta+1)^2}\Omega_5 + \frac{a\varrho_1R_7}{(\delta+1)^2}\Omega_6 + \frac{a\eta_2}{(\delta+1)^2X}\Omega_7 + \frac{a\sqrt{L}\Pi\mathrm{K}_1\sigma}{2(\delta+1)^2}\Omega_8 > \frac{2C_\delta}{\delta+1}H_{44}$.
Finally, the sixth-order minor is

$$H_{66} = -bH_{55} + \frac{a}{(\delta+1)^2}\Omega_9 + \mathrm{bR}_5\Omega_{10} + \frac{b\varrho_1R_1}{\alpha\mathrm{Q}}\Omega_{11} + (1-\mathrm{bX})\eta_2\Omega_{12} + \frac{b\sqrt{L}\Pi\mathrm{K}_1\sigma}{2\alpha\mathrm{Q}}\Omega_{13} > 0$$

for a positive $\Omega_9, \Omega_{10}, \Omega_{11}, \Omega_{12}$ and Ω_{13}.

Conflict of interest

All authors declare no conflicts of interest in this study.

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