



বিদ্যাসাগর বিশ্ববিদ্যালয়
VIDYASAGAR UNIVERSITY

Question Paper

B.A./B.Sc./B.Com. Part-III (1+1+1) Examination 2020

3rd Year (Honours)

Subject: MATHEMATICS

Paper: VI

Full Marks: 80 (Theory)

Time: 4 Hours (Theory)

*Candidates are required to give their answer in their own words as far as practicable.
Questions are of equal value.*

Answer any **one question** from the following:

1. (a) Let A,B,C,D,E,F be the moments and products of inertia with respect to a system of three rectangular axes through O, then show that the moment of inertia of the material system about any line through O, whose direction cosines are l, m, n is

$$Al^2 + Bm^2 + Cn^2 - 2Dmn - 2Enl - 2Flm.$$

- (b) Explain what is meant by the length of simple equivalent pendulum. Define centres of suspension and oscillation. Show that the centres of suspension and oscillation of a compound pendulum are interchangeable.

2. (a) Show that the momental ellipsoid at any point on the rim of a hemisphere is

$$2x^2 + 7(y^2 + z^2) = \frac{15xz}{4} = \text{const}$$



- (b) Find the moment of inertia of a hollow sphere about a diameter.
3. (a) Obtain the necessary and sufficient condition for the equilibrium of a mass of heterogeneous liquid under a given system of forces whose components per unit mass at a point (x,y,z) parallel to the coordinate axes are (X,Y,Z) .
- (b) Find the depth of the centre of pressure of a plane area immersed in a heavy liquid under gravity.
4. (a) Show that the surface of separation of two liquids of different densities which do not mix at rest under gravity is horizontal.
- (b) A plane area is fully immersed in a homogeneous liquid under gravity. If the plane area is lowered vertically in the liquid, show that C.P. approaches towards C.G. and ultimately coincide with the C.G.
5. (a) Using the generating function technique solve the recurrence relation :
- $$a_n = 3a_{n-1} + 2; \forall n \geq 1, a_0 = 2$$
- (b) Prove that in a graph there is even number of vertices of odd degree.
- (c) Show that a complete graph with n vertices consists of $\frac{n(n-1)}{2}$ edges .
6. Write down the basic assumption of prey-predator model. Formulate it analytically and solve it. Interpret the result geometrically. What are the limitations of the model?
7. (a) Explain the Kinematics of a rigid body.
- (b) Find Kinetic Energy of the rigid body with a fixed point.
- (c) Find Angular Moment of the rigid body.
8. (a) State D'Alembert's principle and deduce general equations of motion of a rigid body.
- (b) Prove that the motion of a rigid body about its centre of inertia is the same as it would be of the centre of inertia were fixed and same forces acted on the body
- (c) Show that the rate of change of angular momentum of a rigid body about the axis of rotation is equal to the sum of moments about the same axis of all forces acting on the body.



9. (a) Find the thrust of heavy homogeneous liquid on plane Surface. Hence define the term superincumbent liquid.
- (b) A vessel is in the form of a right circular cone without weight, the vertical angle being $2a$; the vessel filled with liquid and then suspended by a point in the rim; if β be the inclination of the axis of the cone to the vertical, show that $\cot 2\beta = \cot 2a \frac{3}{4} \operatorname{cosec} 2a$
- (c) What happens to the position of the centre of pressure if the plane area lowered infinitely?
10. (a) Discuss the necessary and sufficient conditions for equilibrium of a fluid under the action of external forces of components (X, Y, Z) per unit mass acting at the point (x, y, z) in the fluid. Is this condition are true for irrotational field of force? Justify.
- (b) Show that a homogeneous right circular cone of vertical angle $2a$ cannot float stably with its axis vertical and vertex downwards unless its density as compared with that of the liquid is greater than $\cos^6 a$.
- What is the corresponding result when the vertex is upward?
11. (a) Show that any graph is connected if it has at least $(n-1)$ edges, where n is the number of vertices in the graph.
- (b) What do you mean by Tautology and Contradiction? Give one example of each.
- (c) Define poset. If n be a positive integer and D_n denotes the set of all divisors of n , consider the partial order 'divides' in D_n . Then draw the Hasse diagrams for D_6, D_{24}, D_{30}
12. (a) Show that the mathematical model represent by
- $$\frac{dx}{dt} = x(15 - 5x - 3y), \frac{dy}{dt} = -y(4 - x - y), x, y \geq 0.$$
- has a position of equilibrium. Show also that this position is stable and two species can coexist.
- (b) Write the basic assumption of single species model when birth rate and death rate are constants. Construct the differential equation of the model and solve it. Show that population grows or decays exponentially.